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Black hole entropy in string-generated gravity models

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The Euclidean action and entropy are computed in string-generated gravity models with quadratic curvatures, and used to argue that a negative mass extremal metric is the background for hyperbolic ($k=-1$) black hole spacetimes, k being the curvature constant of the event horizon. The entropy associated with a black hole is always positive for $k=\{0, 1\}$ family. The positivity of energy condition also ensures that the $k=-1$ (extremal) entropy is non-negative.

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The area-entropy law [1] (in Planck units) $\mathcal{S}=A_H/4G$ (where A_H is the area of the event horizon of the black hole and G is the Newton constant) is one of most celebrated results in general relativity. It is known [2] that the black hole entropy is not simply given by one-quarter the area, particularly, if one allows higher curvature corrections to the Einstein action, such as

$$I=\frac{1}{16\pi G}\int d^{n+1}x\,\sqrt{-g}(R-2\Lambda)+\alpha_1\int d^{n+1}x \times\sqrt{-g}(R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}+aR_{\mu\nu}R^{\mu\nu}+bR^2)+\cdots. \quad (1)$$

There are some known reasons to explore black holes in such generalized gravity models. The Gauss-Bonnet (GB) term obtained by setting $a=-4, b=1$, originally motivated by string theory, produces the most general Lagrangian retaining only second-order field equations, and admits exact spherically symmetric solutions in dimensions $n+1>4$ [3]. The action (1) with $a=b=0$, $n=4$ corresponds to an effective AdS_5 (bulk) action, deduced from a heterotic string via heterotic-type I duality [4],

$$I=\frac{N^2}{4\pi^2\ l^3}\int d^5x\sqrt{-g}\left[(R-2\Lambda)+\frac{l^2}{16N}R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}\right], \quad (2)$$

where, using AdS conformal field theory (CFT) duality [5], the coefficient of $(\text{Riemann})^2$ term is fixed as $32\pi G\ \alpha_1 = l^2/8N\equiv\varepsilon$.

One can evaluate leading order corrections to the black hole entropy by finding exact solutions of Einstein equations supplemented by higher curvature (HC) terms, such as a Gauss-Bonnet term or quadratic interactions without a $(\text{Riemann})^2$ term, or by treating HC terms as perturbation about the Einstein gravity. The first approach allows one to study global properties of the solutions with an asymptotically (anti–)de Sitter branch [2,3,6,7]. In this context, a question may be raised as to whether higher derivative gravities can have negative entropy [6,8], in particular, when the curvature length of AdS geometry itself is in the order of HC couplings. In order to address this issue and gain some insight into the problem, it is essential to calculate the total energy. In doing so, we find that the requirement of positivity of energy ensures the positivity of (extremal) black hole entropy.

In this paper, we also answer to the important question of what is the correct ground state to use in hyperbolic anti–de Sitter spacetimes. We reiterate the earlier assertions made by Vanzo [9] and Birmingham [10] (see also [11] for a discussion in the context of the counterterm substraction method) that a negative mass extremal metric is the background for hyperbolic black holes [12].

The action (1) with $a=-4$, $b=1$, admits the exact black hole solution [6,7,13]

$$ds^2=-f(r)dt^2+\frac{dr^2}{f(r)}+r^2\sum_{i=1}^{n-1}h_{ij}dx^idx^j \quad (3)$$

$$f(r)=k+\frac{r^2}{2\alpha}\mp\frac{r^2}{2\alpha}\sqrt{1+\frac{8\alpha\Lambda}{n(n-1)}+\frac{4\alpha\ \mu}{r^n}} \quad (4)$$

where $\alpha\equiv16\pi G(n-2)(n-3)\alpha_1$, μ is a mass parameter, and h_{ij} is the metric of an $(n-1)$-dimensional maximally symmetric space $\mathcal{M}_k^{n-1}$ with curvature $k=0, \pm1$. For a symmetric space $R_{\mu\nu\lambda\rho}=-(g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda})/\ell^2$, the cosmological term is fixed $\Lambda=-n(n-1)/2l^2$, where $l=\ell/\sqrt{1-\alpha/\ell^2}$ is the (effective) curvature radius of AdS bulk geometry. One also identifies the imaginary time of the solution with a period $\beta=4\pi/f'(r_+)$, namely

$$\beta=\frac{4\pi\ r_+l^2\ (r_+^2+2\alpha\ k)}{nr_+^4+(n-2)k\ r_+^2\ l^2+(n-4)\alpha\ k^2\ l^2}, \quad (5)$$

where r_+ is the largest positive root of $f(r)$ [cf., negative root in Eq. (4)], and $1/\beta=T$ is the Hawking temperature of a black hole.

The extremal black holes are defined to have zero temperature, which require a vanishing denominator in Eq. (5). Therefore, for $n=4$, there is an extremal $k=-1$ solution, with a degenerate horizon at $r_+=r_e$, satisfying

$$r_e^2=\frac{l^2}{2}, \quad \mu_e=-\frac{l^2}{4}\left(1-\frac{4\alpha}{l^2}\right), \quad (6)$$

and $\alpha<l^2/4$. Here we need to be more precise. The above solutions are extremal ones only if $\alpha<l^2/4$ holds. Because, in particular, for the coupling $\alpha=l^2/4$, one obtains $\beta=8\pi r_+ l^2/[2nr_+^2+(n-4)kl^2]$, and hence the Hawking temperature is finite, namely, $T=r_+/\pi l^2$, when $n=4$, independent of the curvature k of the horizon. That is to say, the extremal Hawking temperature can be zero only for the coupling $\alpha<l^2/4$ [13]. To present a better picture, we need to consider the $n>4$ case. With $k=-1$, the $T=0$ ($\beta=\infty$) condition yields

$$r_{c,\ e}^2=\left(\frac{n-2}{2n}\right)l^2\left(1\mp\sqrt{1-\frac{4n(n-4)}{(n-2)^2}\,\frac{\alpha}{l^2}}\right) \tag{7}$$

$$\mu_{c,\ e}=\left(\frac{2r_{c,\ e}^{n-2}}{n-4}\right)\left[\frac{2}{n}\pm\sqrt{\left(\frac{n-2}{n}\right)^2-\frac{4(n-4)}{n}\,\frac{\alpha}{l^2}}\right]. \tag{8}$$

For $\alpha=0$, the critical horizon $r_c(<r_e)$ given by the negative root of Eq. (7) coincides with the singularity at $r=0$, so the space-time region $r<r_e$ has an internal infinity. With $\alpha>0$, we can have non-degenerate horizons with hyperbolic geometry. Moreover, with $\alpha=l^2/4$, one has $r_+=r_c=l\sqrt{(n-4)/2n}$ and hence $T=n(r_+^2-r_c^2)/4\pi r_+ l^2=0$, which is of course not a massless [Bogomol'nyi-Prasad-Sommerfield (BPS)] state, since $\mu_c>0$. This corresponds to a particular solution studied in [14], where the coupling α' is fixed in the starting action using $8\alpha\Lambda+n(n-1)=0$. Notice that, for $\alpha=l^2/4$, $\mu_e=0$ at $r_+=r_e=l/\sqrt{2}$ but $T\neq0$. A clear message is that only for $\mu_e<0$ (or $\mu_c>0$) background one can consistently set $T=0$. The possible backgrounds are

$$n=4:\ \ \mu_e=0,\quad T=\frac{1}{\sqrt{2}\,\pi l}\quad \text{or } \mu_e<0,\quad T=0,$$

$$n>4:\ \ \mu_e=0,\quad T>0\ \text{ or }\ \mu_c>0,\quad T=0,$$

$$\text{or }\ \mu_e<0,\quad T=0.$$

It would be natural to call "ground state" the state with zero temperature. We find that only a negative mass extremal state can be stable under gravitational (tensor) perturbations. So a massless state may not be the ground state for the $k=-1$ horizon, as expected in [9,10,15].

The on-shell Gauss-Bonnet gravity action reads

$$I=\frac{1}{16\pi G_{n+1}}\int d^{n+1}x\,\sqrt{-g}\left(-\frac{2R}{n-3}+\frac{8\Lambda}{n-3}\right). \tag{9}$$

It is known that the AdS space [16] and the Horowitz-Myers soliton [17] are the appropriate backgrounds, respectively, for spherical ($k=1$) and toroidal ($k=0$) horizons. For $k=0$, a zero mass ground state is still legitimate, and is an acceptable background [9,10]. For $k=-1$, by matching the asymptotic geometries between extremal and asymptotically locally AdS metrics, one subtracts a non-zero mass extremal background [10], restricting attention to the region $r\geqslant r_e$ for the background and $r\geqslant r_+$ for the black hole. The Euclideanized action, valid for $k=0,\ \pm1$, is thus evaluated to be

$$\hat{I}=-\frac{(n-1)V_{n-1}\,r_+^{n-4}\,\beta}{16\pi G_{n+1}\,(n-3)}\left[(kr_+^2-\alpha\,k^2)+\frac{3r_+^4}{l^2}\right]$$
$$+\frac{V_{n-1}r_+^{n-1}}{2(n-3)G_{n+1}}-\frac{(n-1)V_{n-1}\,\beta}{16\pi G_{n+1}}\,\mu_e, \tag{10}$$

where $V_{n-1}=\int d^{n-1}x\,\sqrt{h}$. One reads off the free energy from $F=\hat{I}/\beta$. When $\alpha=0$, $k=-1$, there is no phase transition since the black hole dominates over μ_e background for all temperatures. Typically, a massless state at $\alpha=l^2/4>0$ has an initial positive free energy in $n=4$ but zero free energy in $n=6$, so, for $\alpha>0$ solutions, the behavior of Hawking-Page phase transition could depend on spacetime dimensions, unlike in the $\alpha=0$ case [16,18].

Since μ_e is temperature (or horizon r_+) independent, the black hole entropy takes a remarkably simple form

$$\mathcal{S}=\beta^2\,\frac{\partial F}{\partial\beta}=\frac{A_H}{4G_{n+1}}\left[1+\frac{(n-1)}{(n-3)}\,\frac{2\alpha k}{r_+^2}\right], \tag{11}$$

where $A_H=V_{n-1}\,r_+^{n-1}$. This derivation is essentially an application of Eq. (10) and second law of black hole thermodynamics. So, conceptually, it is fundamentally different from the calculation in [13] where entropy comes from first law. Equation (11) is the correct entropy formula even in a flat spacetime ($\Lambda=0$) [2], so the cosmological constant on the AdS boundary is not dynamical. As a result, the central charge of an effective theory with a GB term allows one to compute entropy without breaking Virasoro algebra near the horizon [19]. The entropy flow

$$d\mathcal{S}=\frac{(n-1)A}{4G_{n+1}r_+}\left(1+\frac{2\alpha k}{r_+^2}\right), \tag{12}$$

is always positive, because $r_+^2+2\alpha k\geqslant0$ should hold for black hole interpretation [7], and satisfies a generalized second law [1]. Moreover, since

$$T\mathcal{S}=\frac{(n-1)V_{n-1}r_+^{n-4}}{16\pi G_{n+1}(n-3)}\left[(n-2)kr_+^2+(n-4)\alpha k^2+\frac{nr_+^4}{l^2}\right]$$
$$-\frac{V_{n-1}r_+^{n-1}}{16\pi G_{n+1}}\,\frac{8\pi T}{n-3}, \tag{13}$$

one readily evaluates the thermodynamic energy to be

$$E=T\mathcal{S}+F=M-\frac{(n-1)V_{n-1}}{16\pi G_{n+1}}\,\mu_e=M-M_e,$$

$$M=\frac{(n-1)V_{n-1}}{16\pi G_{n+1}}\left(kr_+^{n-2}+\frac{r_+^n}{l^2}+\alpha k^2 r_+^{n-4}\right). \tag{14}$$

For $k=1$, since $M_e=0$, one has $E=M$. For $k=-1$, since $M_e<0$, $E\neq M$, in general. It is quite interesting that, for $k=-1$, $E=0$ at $r_+=r_e$, and $E>0$ otherwise. Consider for concreteness the $n=4$ case. Then, one has

$$E=\frac{3V_3}{16\pi G}\,\mu+\frac{3l^2V_3}{64\pi G}\left(1-\frac{4\alpha}{l^2}\right). \tag{15}$$

This energy is vanishing at the extremal state, and also in Nariai limit $\mu=\alpha-l^2/4$. As in the de Sitter case [22], the Nariai solution is not the ground state in $n\neq 4$.

The black hole entropy (11) is always positive for the curvature $k=0$, 1. However, for $k=-1$, one has

$$\mathcal{S}=\frac{V_{3,k=-1}\,r_+^3}{4G_5}\left(1-\frac{6\alpha}{r_+^2}\right)\Rightarrow\mathcal{S}_e=\frac{V_3}{G_5}\,\frac{l^3}{2^{7/2}}\left(1-\frac{12\alpha}{l^2}\right). \tag{16}$$

Thus, in particular, when one approaches a massless state at $\alpha=l^2/4$, the extremal entropy becomes negative. This is of course not an encouraging situation, because, as a microscopic interpretation, the black hole entropy is the logarithm of the number of (quantum) states and should be positive. It is expected that additional higher order corrections, like that of R^4 terms, might cure this problem, so that a full theory will yield only positive (extremal) entropy. One also notes that, for the $\alpha=0$ case, the $k=-1$ extremal ground state has positive entropy [11]

$$\mathcal{S}_e=\frac{V_{n-1}}{G_{n+1}}\,\frac{l^{n-1}}{2^{(n+3)/2}}. \tag{17}$$

These results further provide a hint that a massless extremal state is simply not allowed as a ground state.

As the first plot in Fig. 1 shows, the small horizon regime $r<r_e$ has a single branch for $\alpha=l^2/4$ and two branches for $\alpha<l^2/4$. The first branch (cusp) on the left, which might have negative specific heat, has no black hole interpretation since this region is not allowed due to a constraint $r_+^2>2\alpha$. Here we should note that, when $k=-1$, $n=4$, for the coupling $\alpha=l^2/12$, the Euclidean period β is negative in the range $\frac{1}{6}<r_+^2<\frac{1}{2}$, i.e., $0.408<r_+<0.707$. So the Hawking temperature, which is a non-negative entity, should be defined as $T=|\beta^{-1}|$. That is, in the range $0.408<r_+<0.707$, the specific heat must be defined by $C=-\beta^2\,dE/(d(-\beta))$. As a result, the second cusp in the first plot of Fig. 1 should be mirror reflected, and hence can have a positive specific heat. Nevertheless, for the coupling $\alpha=l^2/4$, the Euclidean period β is always positive, so the formula $C=-\beta^2\,dE/(d\beta)$ is still effective. For this particular coupling, the specific heat could be negative, which might signal the instability of a massless state. Because the energy condition $E\geqslant 0$ always holds, the black holes of size of the extremal state or bigger than this have zero or positive specific heat, and the corresponding solutions are thermodynamically stable and globally preferred.

It is interesting that the minimum of the energy is also the minimum of the temperature. As a result, the ratio dE/dT is well behaved even if $k=1$, which should be contrasted with the result in Einstein gravity ($\alpha=0$). This might show the emergence of a stable branch of small spherical black holes, and similar result was realized by Caldarelli and Klemm in [20], where a detailed treatment of M theory or stringy corrections, specifically, the $\mathcal{O}(\alpha'^3)$ corrections of type IIB string theory, to black hole thermodynamics is presented. It has been shown that [20] the leading stringy or M-theory corrections do not give rise to any phase transition for flat and hyperbolic horizons, although to a quotient of hyperbolic space H^{n-1}/Γ there may arise new phase transitions. Further elaboration and related discussion upon this issue appear in [21].

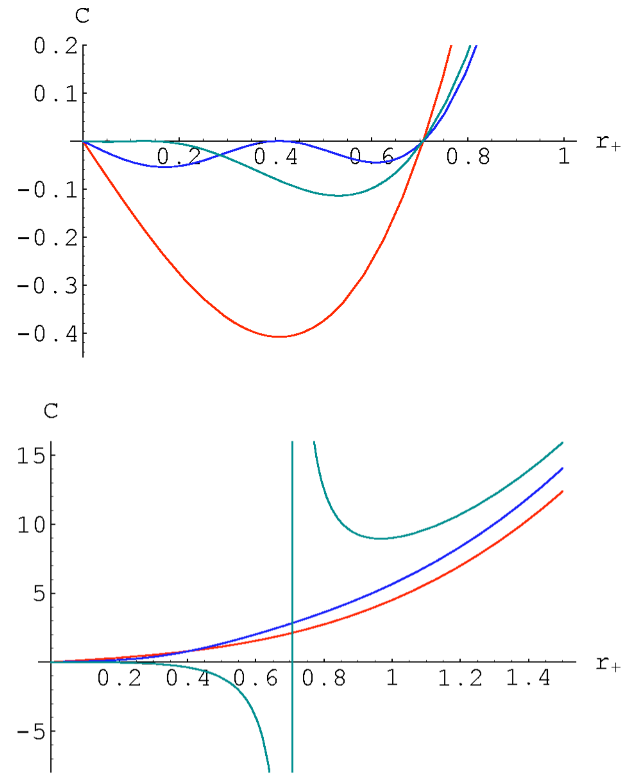

FIG. 1. The specific heat ($C=dE/dT$) vs horizon radii. The parameters are fixed as $l=1$, $V_{n-1}/4G_{n+1}=1$, $n=4$; (a) $k=-1$ (upper plot): $\alpha=1/4$ (big single cusp), $\alpha=1/12$ (two cusps), and $\alpha=1/120$ (small single cusp). (b) $k=1$ (lower plot): the curve with $\alpha=0$ develops singularity at $r_+=1/\sqrt{2}$, so a small (large) black hole has negative (positive) specific heat, and two other curves correspond to $\alpha=1/12$ and $\alpha=1/4$ (up to down).

Given the importance of Gauss-Bonnet corrections to Einstein gravity, the extremal entropy is non-negative only if $12\alpha<l^2$. This constraint also enforces the positivity of energy for the $k=1$ case. Following [6,22], we may calculate the total mass (quasi-local energy) of $k=+1$ Schwarzschild anti–de Sitter spacetime using the surface energy-momentum tensor. In $n+1=5$, we find

$$E=\frac{3V_3\,l^2}{16\pi G}\left(1-\frac{12\alpha}{l^2}\right)\left(\frac{1}{4}+\frac{\mu}{l^2}\right). \tag{18}$$

In using a relation $d\mathcal{S}=\beta\,dE$, we arrive at

$$\mathcal{S}=\int\frac{dr_+}{T}\,\frac{dE}{d\mu}\,\frac{d\mu}{dr_+}=\frac{V_3}{4Gl^2}(r_+^3+6\alpha\,r_+)(l^2-12\alpha)+\mathcal{S}_0. \tag{19}$$

This entropy is non-negative when $\alpha < l^2/12$, since $\mathcal{S}_0 = 0$ at $r_+ = 0$. It is worth noting that the positivity of energy and entropy in the $k = +1$ case also ensures that the $k = -1$ extremal entropy is non-negative.

Next we consider the action (2). The metric solution that solves the field equations, to the order $\mathcal{O}(\varepsilon)$, is

$$f(r) = k - \frac{m}{r^2} + \frac{r^2}{l^2} + \frac{\varepsilon m^2}{r^6}, \tag{20}$$

where l^2 is related to Λ via $l^2(\Lambda\, l^2 + 6) = 2\,\varepsilon$. Using $f(r_+) = 0$, we express m as a function of horizon radius:

$$m = \left(k + \frac{r_+^2}{l^2}\right)\left[r_+^2 + \varepsilon\left(k + \frac{r_+^2}{l^2}\right)\right] \equiv \frac{4\pi^2 l^3}{N^2}\,\frac{M}{3V_3}, \tag{21}$$

where r_+ is the largest positive root of $f(r)$ and M is the black hole mass. The inverse Hawking temperature is

$$\beta = \frac{1}{T} = \frac{2\pi r_+^3 l^4}{r_+^2 l^2 (k l^2 + 2 r_+^2) - 2\varepsilon (k l^2 + r_+^2)^2}. \tag{22}$$

The extremal $k = -1$ solution, implied by $T = 0$, reads

$$r_e^2 \simeq \frac{l^2}{2}\left(1 + \frac{\varepsilon}{l^2}\right), \quad m_e \simeq -\frac{l^2}{4}\left(1 - \frac{\varepsilon}{l^2}\right). \tag{23}$$

Using these as background values for $k = -1$, we obtain an Euclideanized action valid for $k = 0, \pm 1$ to be [23,24]

$$\hat{I} = \frac{V_3 N^2}{4\pi^2 l^3}\left[\left(m - \frac{2 r_+^4}{l^2}\right)\left(1 - \frac{2\varepsilon}{l^2}\right) - \frac{6\varepsilon m^2}{r_+^4} + \frac{3 l^2}{4}\left(1 + \frac{\varepsilon}{l^2}\right)\right]. \tag{24}$$

The last expression, independent of r_+, will be in effect only to the $k = -1$ case. As usual, free energy is defined by $\hat{I} = \beta F$. The resulting entropy is

$$\mathcal{S} = \beta^2 \frac{\partial F}{\partial \beta} = \frac{V_3\, r_+^3}{4}\,\frac{4N^2}{\pi l^3}\left[1 + \frac{1}{4N}\,\frac{2 r_+^2 + 3 k l^2}{r_+^2}\right]. \tag{25}$$

This entropy is essentially positive in the limit $r_+ \gg l$. It may be negative for $k = -1$ when $r_+^2 < 3\, l^2/2$, but this limit is not allowed due to the energy condition $E \geqslant 0$. In the large N limit, Eq. (25) approximates to usual form $\mathcal{S} = A_H/4G$. So one can expect that for large black holes the asymptotic regions feel only minor corrections due to the higher curvature terms. The extremal entropy

$$\mathcal{S}_e = \frac{V_3}{4G}\,\frac{l^3}{2\sqrt{2}}\left(1 - \frac{1}{N}\right) \tag{26}$$

is positive since $N > 1$. The thermodynamic energy is

$$E = \frac{\partial \hat{I}}{\partial \beta} = F + T\mathcal{S} = M + E_k,$$

$$E_k = \frac{3 V_3 N^2}{4\pi^2 l^3}\left[\frac{2\varepsilon r_+^4}{l^4}\left(1 + \frac{k l^2}{r_+^2}\right) + \frac{k^2 l^2}{4}\left(1 + \frac{\varepsilon}{l^2}\right)\right]. \tag{27}$$

One reads M from Eq. (21). The specific heat $C = \partial E/\partial T$ is

$$C = \frac{3 V_3 r_+^3 N^2}{\pi l^3}\left[\frac{2 r_+^2 (l^2 + 3\varepsilon)}{l^2 (2 r_+^2 - k l^2)} + \frac{k(l^2 + 4\varepsilon)}{2 r_+^2 - k l^2} + \frac{2\varepsilon (l^2 + 2 k r_+^2)(3 k^2 l^4 + 2 k r_+^2 l^2 - r_+^4)}{r_+^2 l^2 (l^2 - 2 k r_+^2)(2 r_+^2 - k l^2)}\right]. \tag{28}$$

A pleasing result is that the energy and specific heat are vanishing at the extremal state defined by Eq. (23), an important hint that the extremal state is the ground state. For $k = 1$, there is a discontinuity in specific heat at $r_+ = l/\sqrt{2}$, even if $\varepsilon > 0$. This is partly because the solutions are only perturbative and we have retained the terms only linear in ε. In the $k = -1$ case, however, the solutions are well behaved, for example, the specific heat and entropy are positive when $r_+ > r_e$. A difference from the $\varepsilon = 0$ case is that now a small size black hole has a positive specific heat at finite coupling $3 < N < \infty$.

We end with a few remarks and future problems.

We have calculated leading order curvature corrections to the black hole entropy with horizons $k = 0, \pm 1$. In general, the entropy is not obtained by evaluating the horizon area of the unperturbed solution divided by $4G$. It is encouraging that the formulas (11), (25) perfectly match with the entropies calculated using Wald's covariant approach [25], where the entropy is (unambiguously) determined by a local geometric expression at the horizon. Presumably, these results provide some elegant test of our knowledge of entropy in string theory, for the higher curvature terms as the Gauss-Bonnet invariant and/or $R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho}$ interaction arise in most string theory as leading α' corrections.

In general, the hyperbolic AdS black hole with zero (extremal) mass is not stable as a supersymmetric background. The stability of a hyperbolic horizon is therefore an important issue in dimensions $n + 1 > 4$, which might be essential for a non-supersymmetric extension of AdS-CFT correspondence. We find that a negative mass $k=-1$ extremal background, which has the lowest energy configuration in its asymptotic class, is stable under gravitational perturbations when $\alpha/l^2 \ll 1$, and the potential is bounded from below (work in preparation). It would be interesting in this case to investigate the thermal phase structures and conformal behavior at infinity by coupling the theory with scalars.

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